DECISION PROCEDURE FOR TRACE EQUIVALENCE

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CONTEXT

- Cryptographic protocols

Most communications take place over a public network

Cryptographic protocols

- small programs designed to secure communication (e.g. secrecy)
- use cryptographic primitives (e.g. encryption, signature)

It important to ensure their security

CONTEXT

- Reliable cryptography
- Correct specification
- Implementation satisfying the specification

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- Secrecy, Authentication, ...

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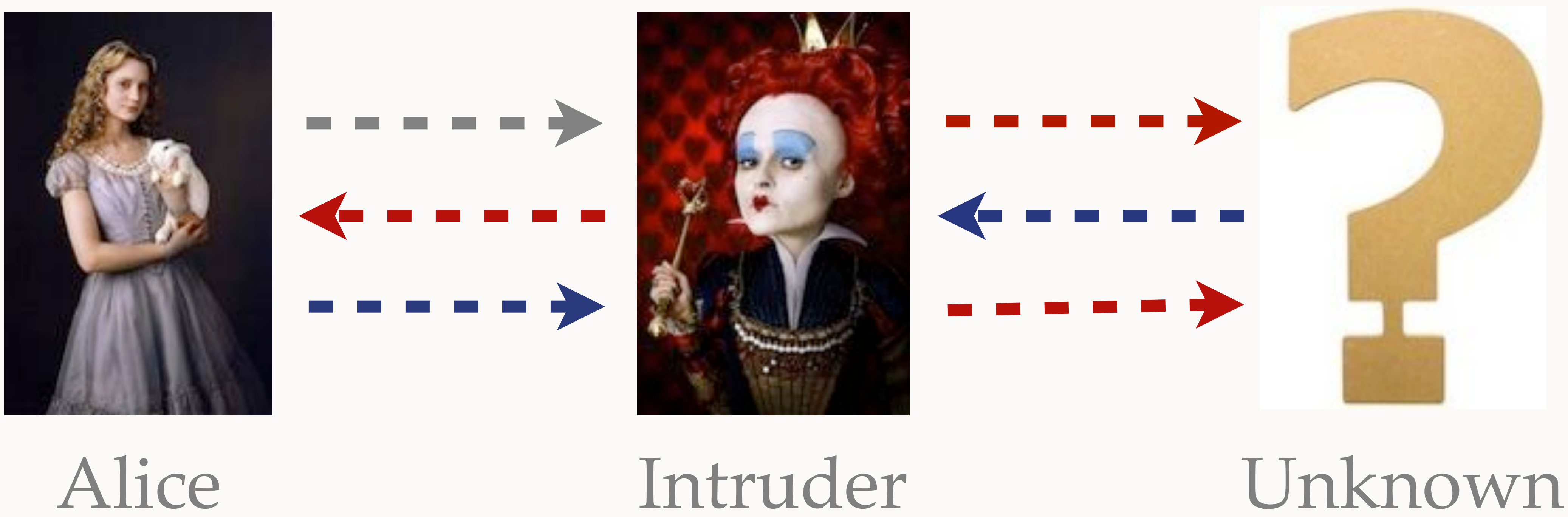
- Secrecy, Authentication, ...

Equivalence properties

- Anonymity, Privacy, Receipt-Freeness, ...

CONTEXT

- Equivalence properties : strong secret, anonymity,...



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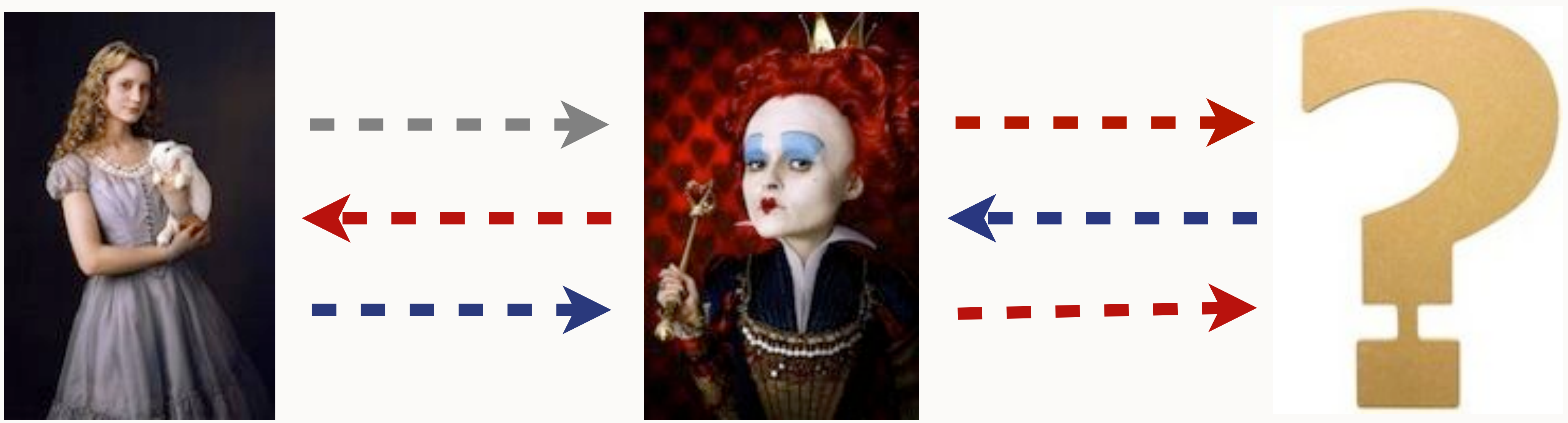
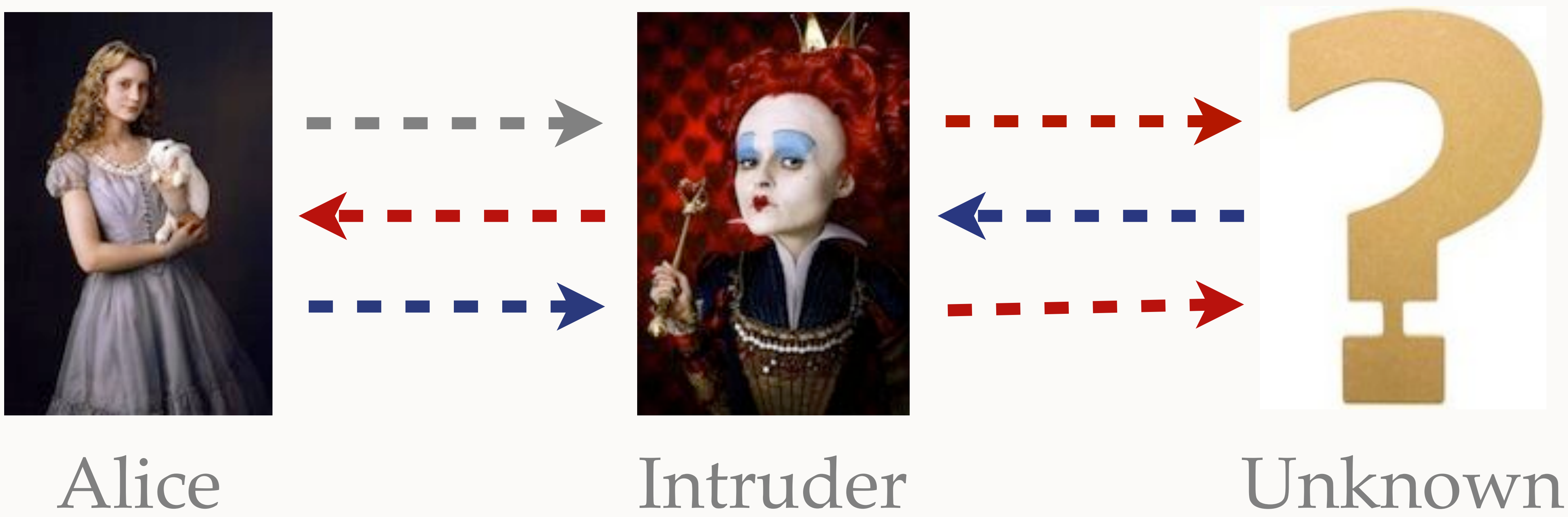


Alice Intruder Unknown

Can the intruder deduce the unknown's identity ?

CONTEXT

- Equivalence properties : strong secret, anonymity,...



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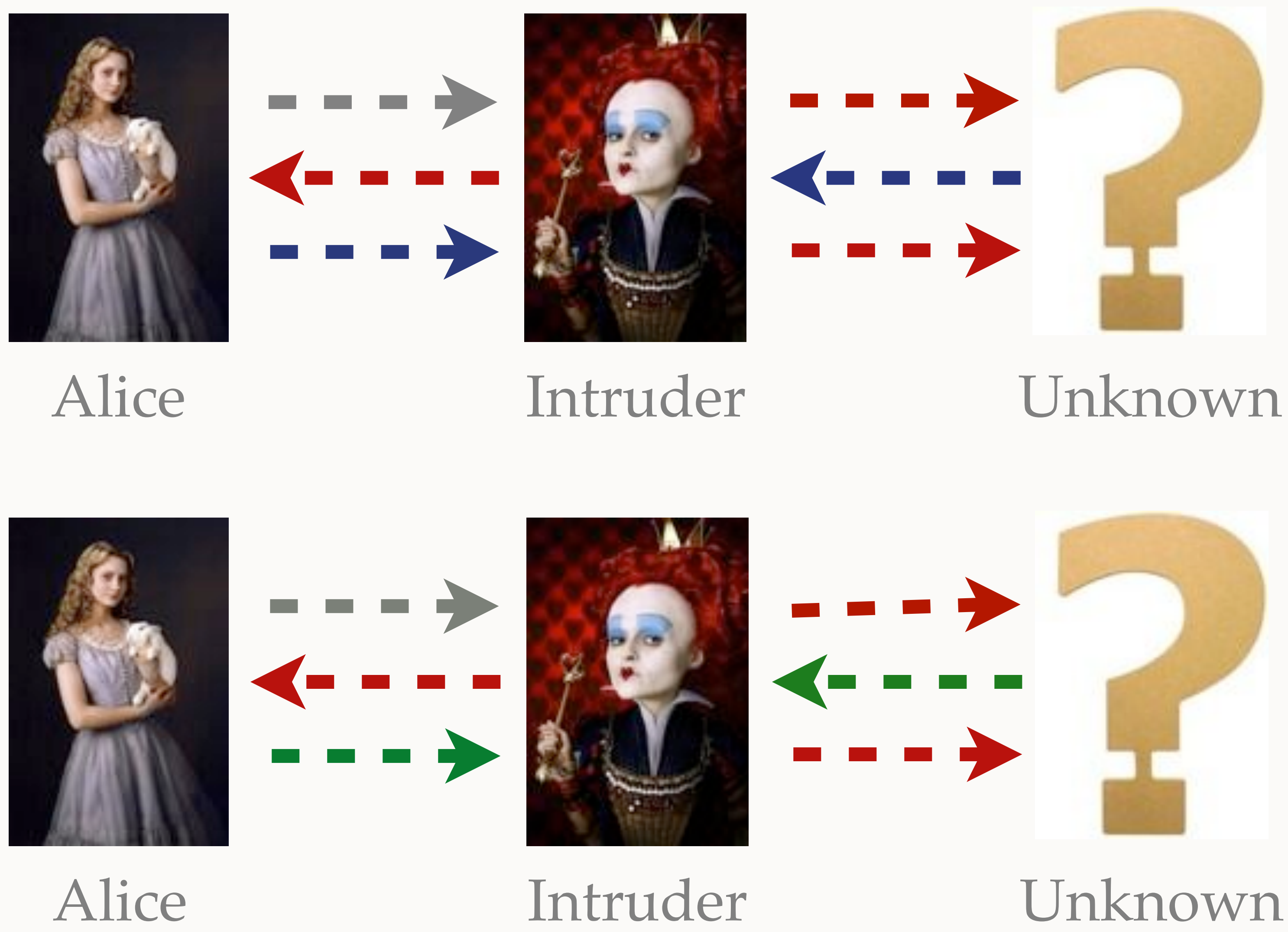

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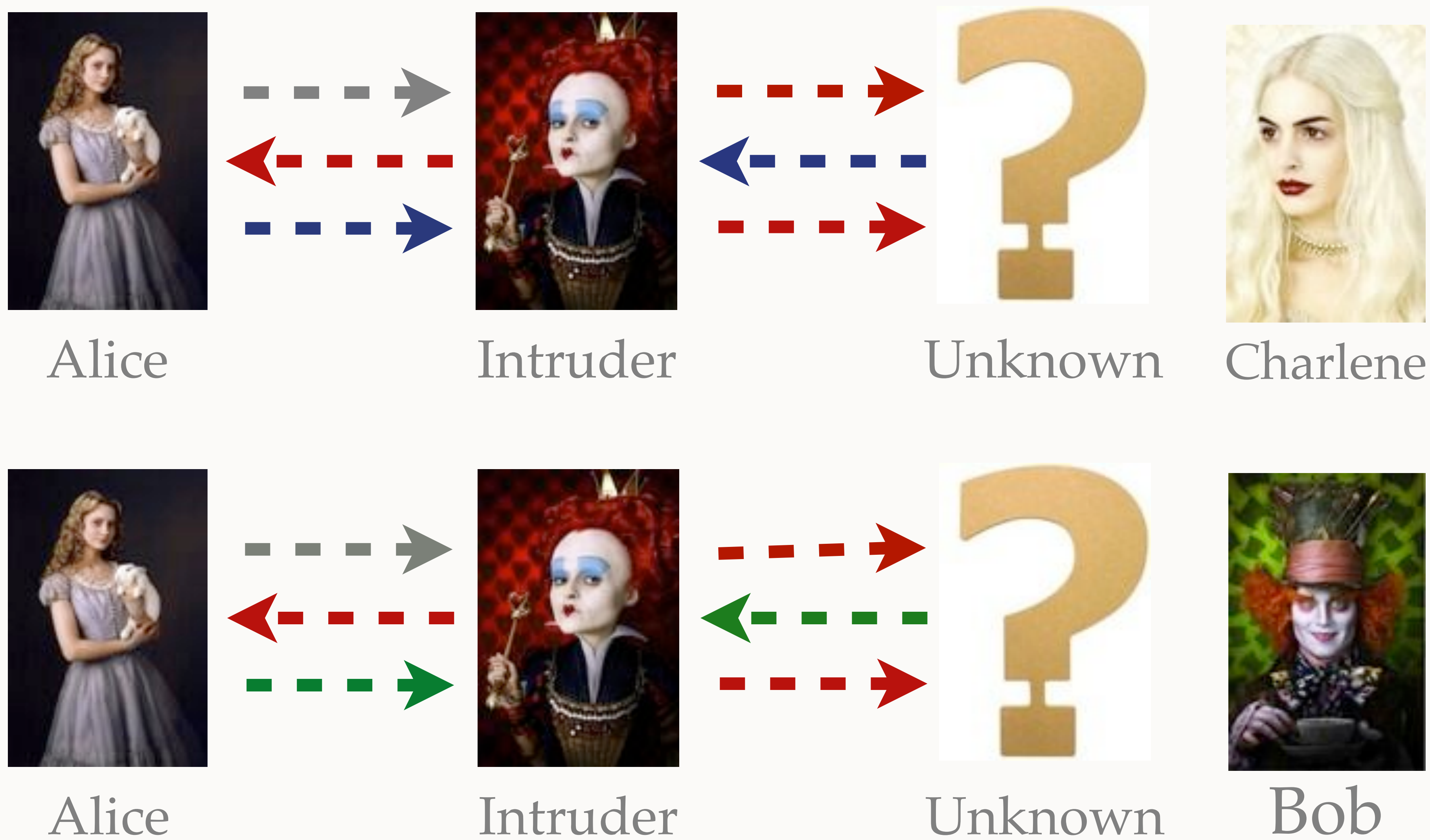

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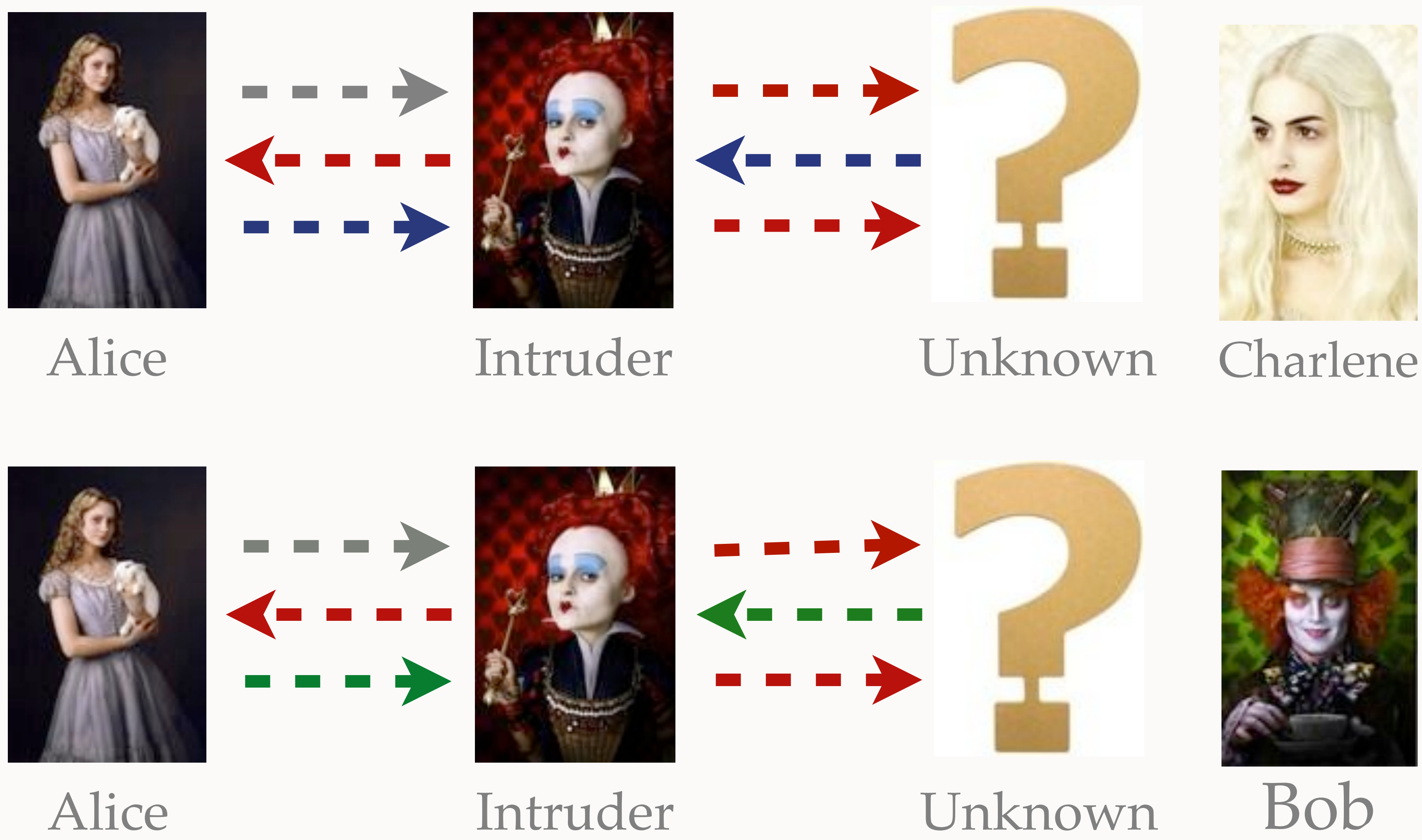
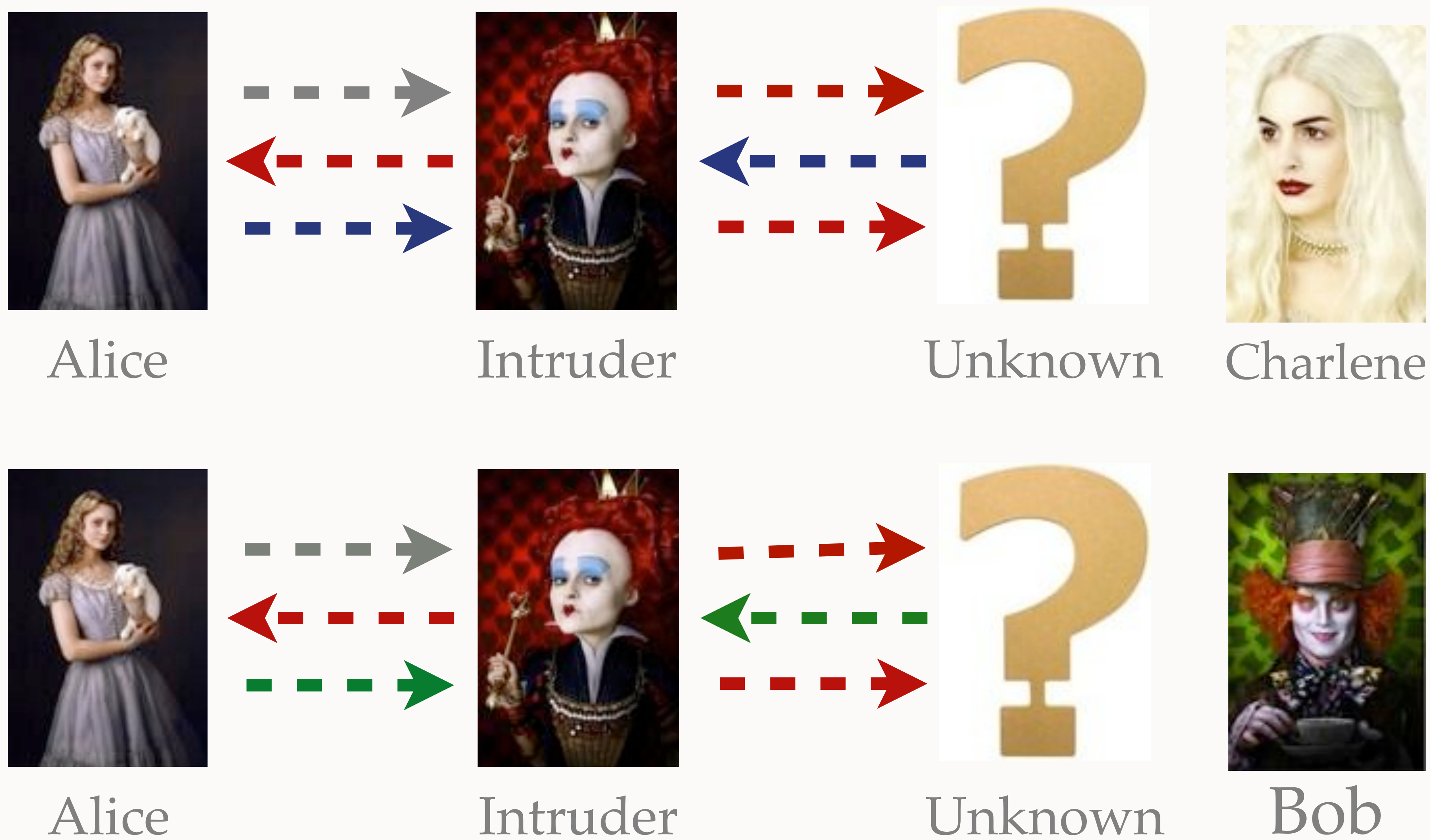


Can the intruder distinguish the two situations ?

CONTEXT

- Equivalence properties : strong secret, anonymity,...



Trace Equivalence

PREVIOUS WORKS

Most of the previous works focus on stronger equivalence

- A. Tiu and J. E. Dawson. *Automating open bisimulation checking for the spi calculus.*
- M. Baudet. *Sécurité des protocoles cryptographiques : aspects logiques et calculatoires.* Phd thesis
- B. Blanchet, M. Abadi, and C. Fournet. *Automated verification of selected equivalences for security protocols.*
  - ➡ Tool : B. Blanchet, *ProVerif*

Trace equivalence for simple processes without else branches

- V. Cortier and S. Delaune. *A method for proving observational equivalence.*

MOTIVATION

- Example

Two problematic examples :

- e-passport protocols : M. Arapinis, T. Chothia, E. Ritter, and M. Ryan. *Analysing unlinkability and anonymity using the applied pi calculus.*
- private authentication protocol : M. Abadi and C. Fournet. *Private authentication. Theoretical Computer Science.*

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Alice

$$\{\langle N_a, pk(k_A)\rangle\}_{pk(k_B)}$$

Bob

MOTIVATION

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$pk(k_A)?$

Bob

MOTIVATION

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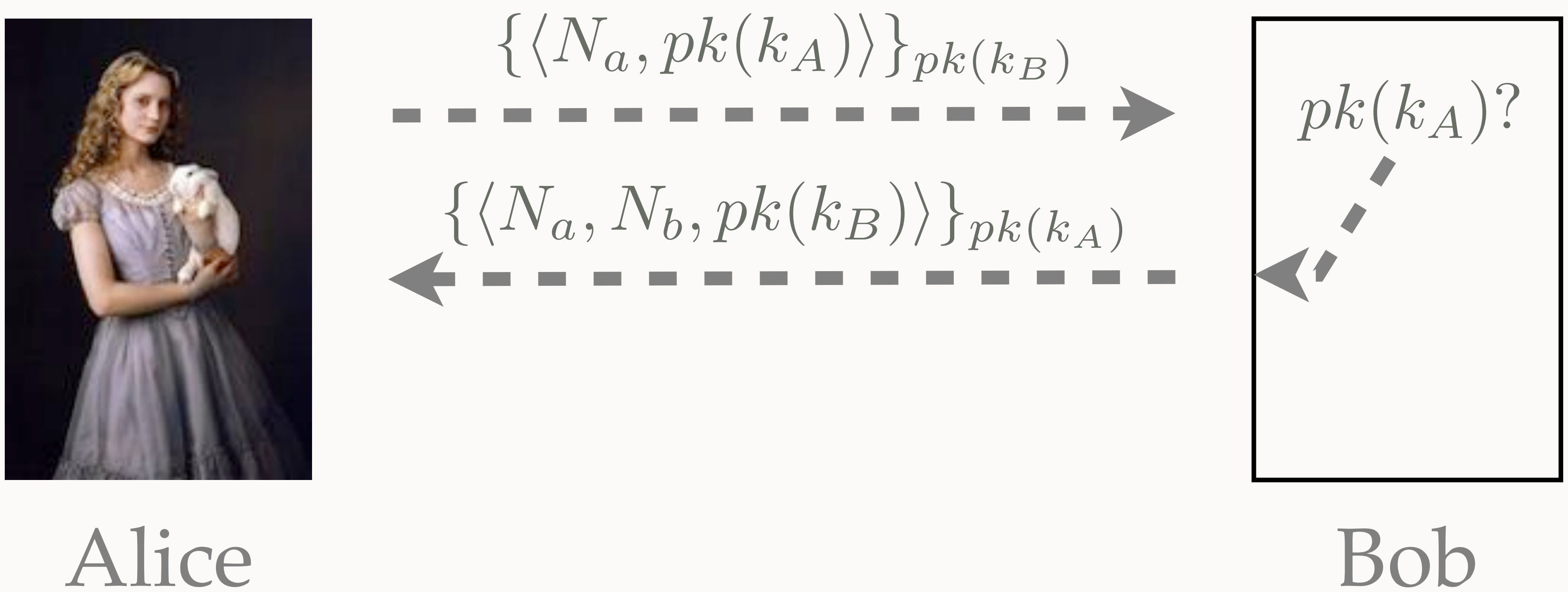

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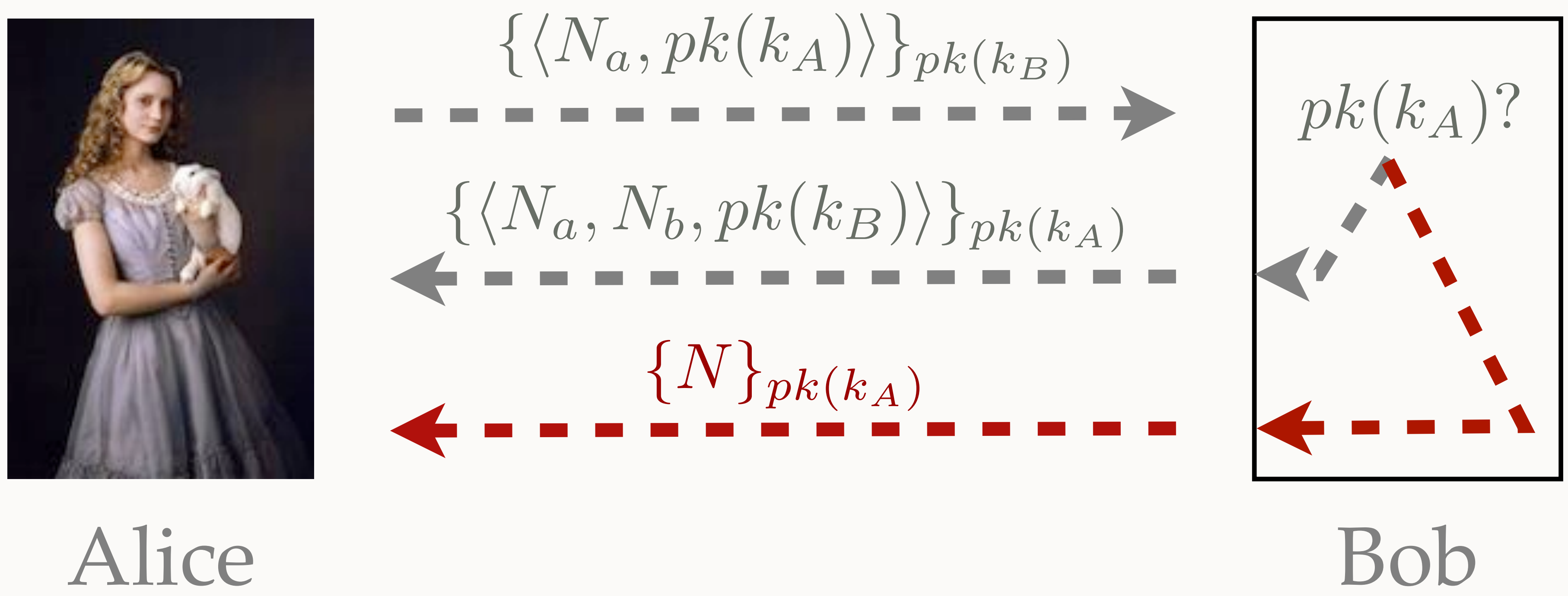

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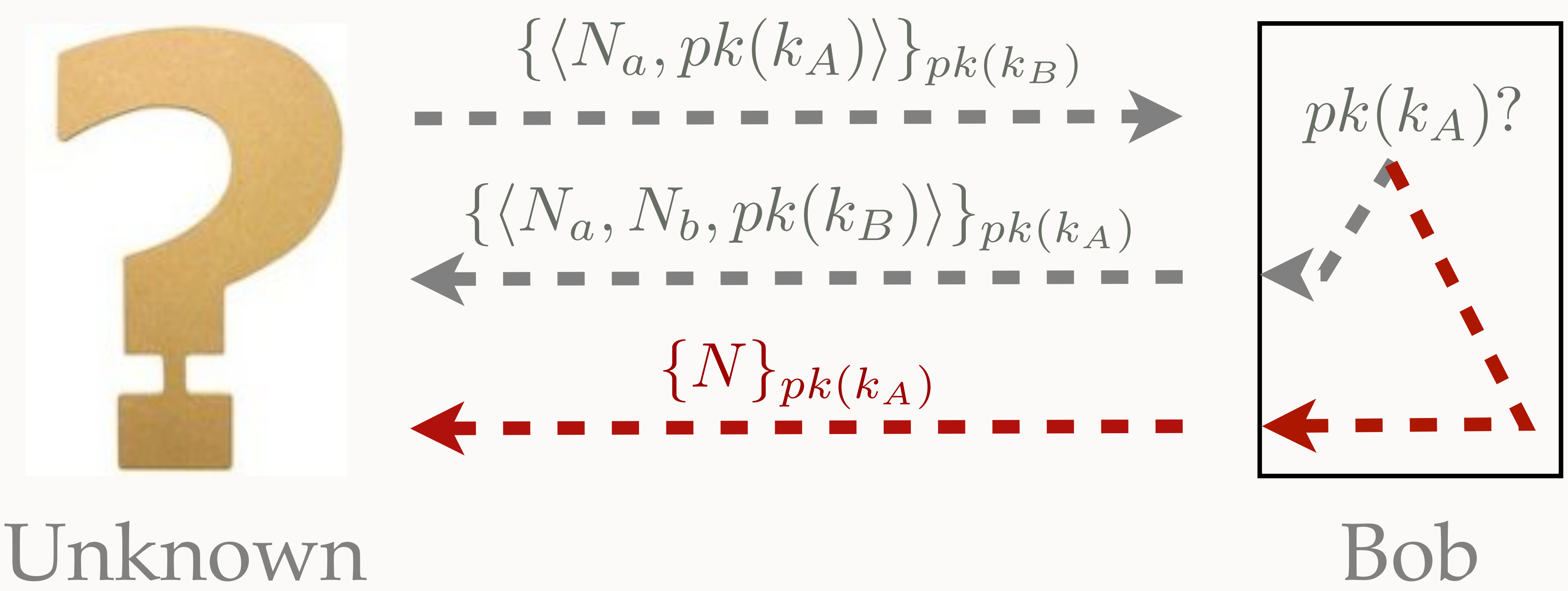


MOTIVATION

- Example

Alice

Intruder

Bob

Charlene

Intruder

Bob

MOTIVATION

- Example

Alice

Bob

Charlene

Bob

MOTIVATION

- Example

$\{\langle N_a, pk(k_A)\rangle\}_{pk(k_B)}$

Alice

Bob

Charlene

Bob

MOTIVATION

- Example

$\{\langle N_a, pk(k_A)\rangle\}_{pk(k_B)}$

$\{\langle x, y\rangle\}_{pk(k_B)}$

Alice

Bob

Charlene

Bob

MOTIVATION

- Example

$\{\langle N_a, pk(k_A)\rangle\}_{pk(k_B)}$ ⇢

$\{\langle x, y\rangle\}_{pk(k_B)}$ ⇢

Alice

$pk(k_A) = y$

Bob

Charlene

Bob

MOTIVATION

- Example

Alice

$\{\langle N_a, pk(k_A)\rangle\}_{pk(k_B)}$

$\{\langle x, y\rangle\}_{pk(k_B)}$

$pk(k_A) = y$

$\{\langle x, N_b, pk(k_B)\rangle\}_y$

Bob

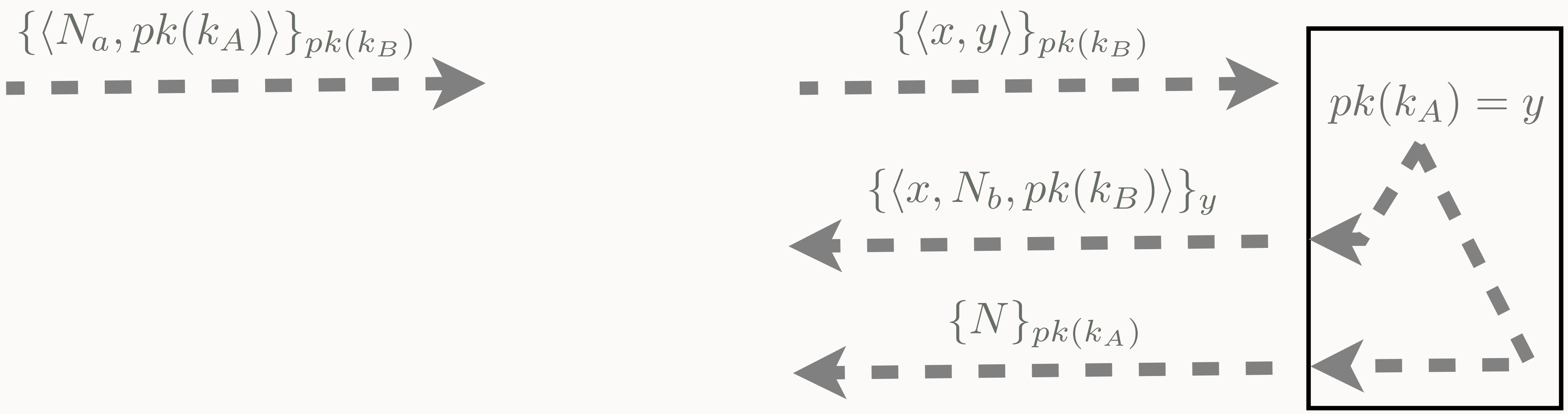
Charlene

Bob

MOTIVATION

- Example

Alice



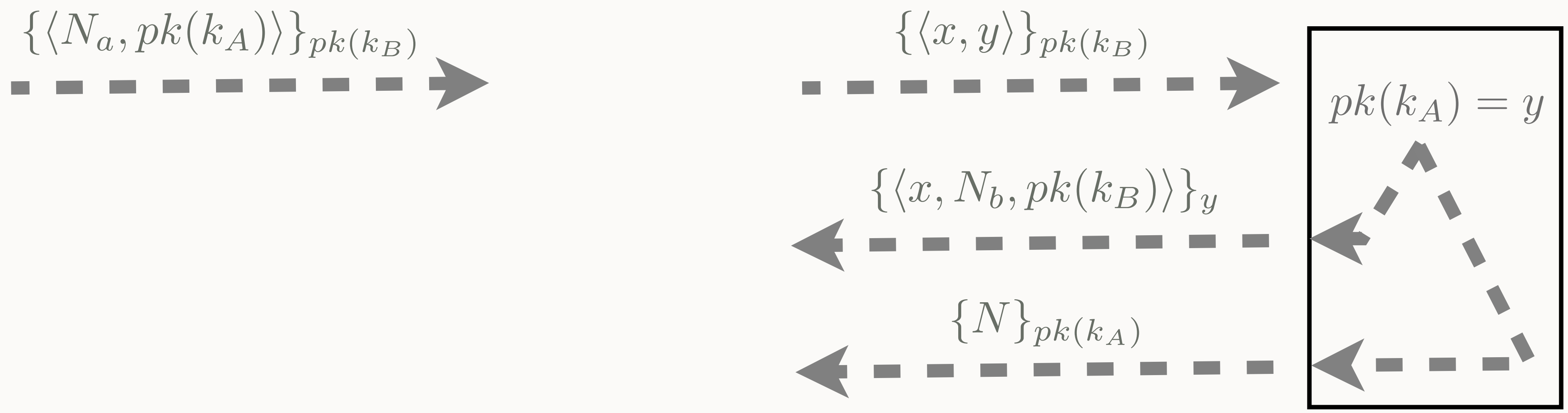
Bob

Charlene

Bob

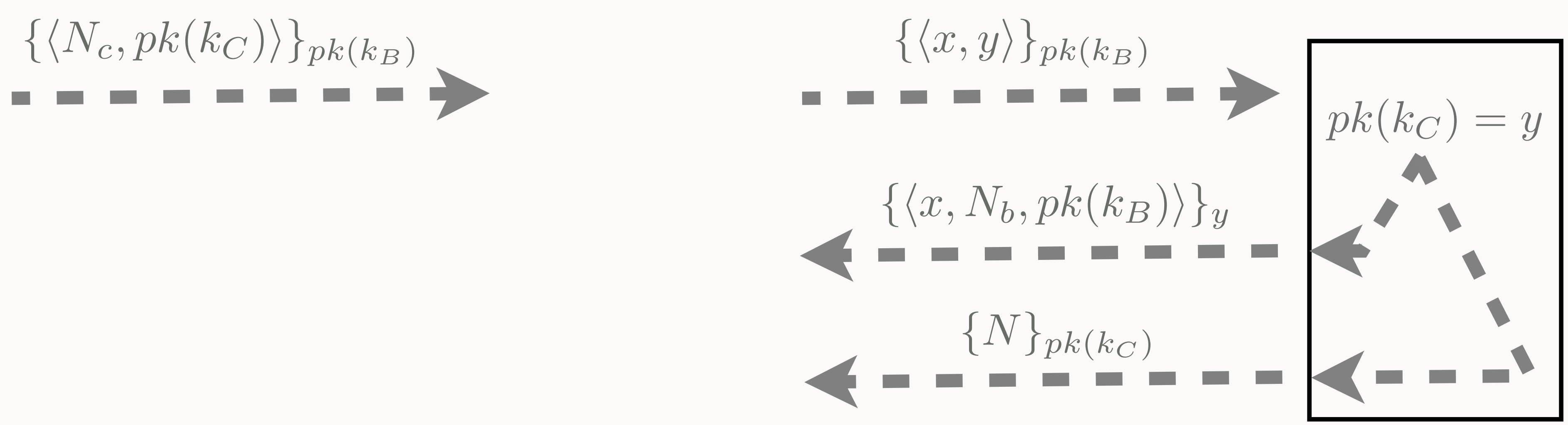
MOTIVATION

- Example



Alice

Bob

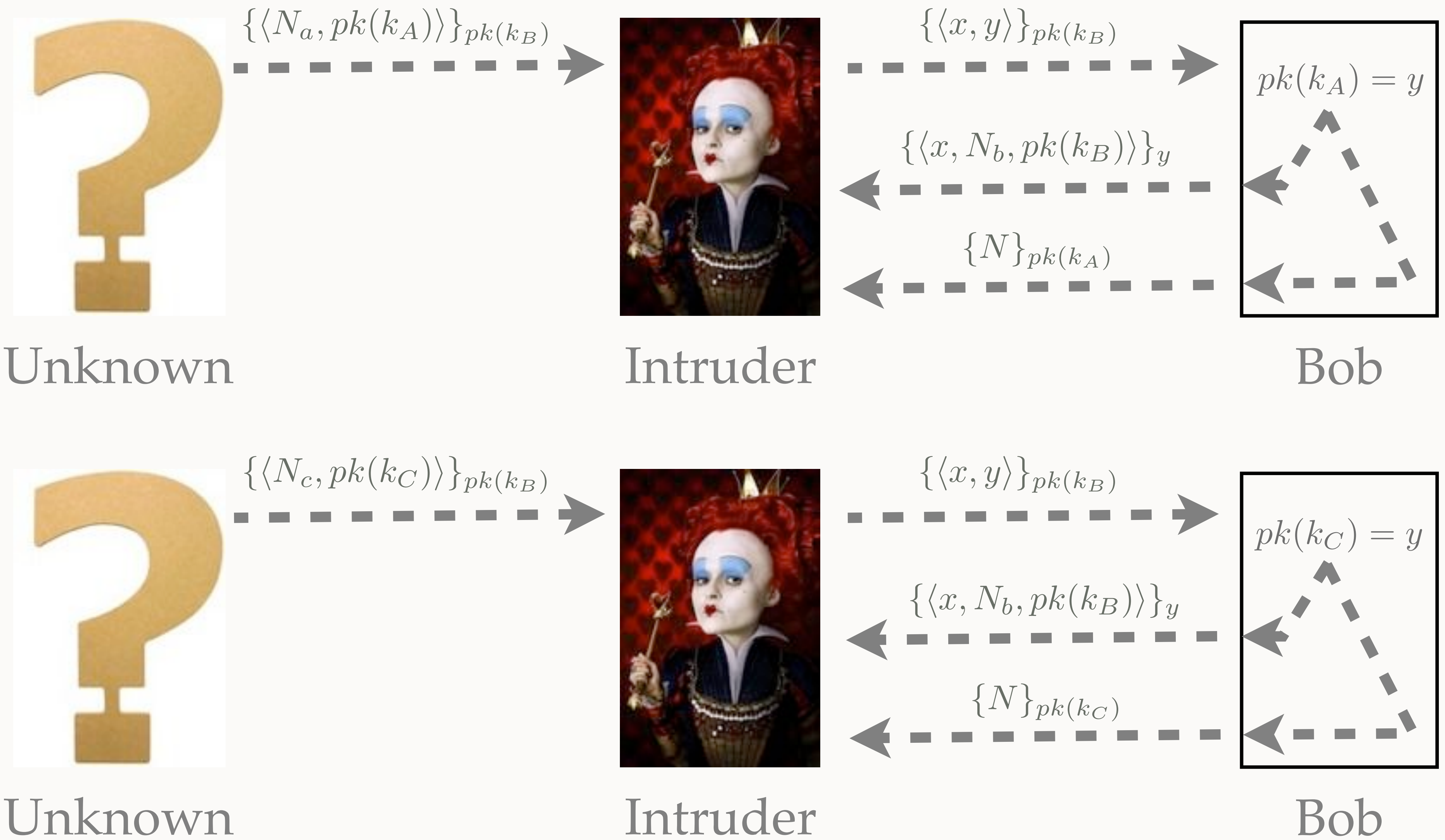


Charlene

Bob

MOTIVATION

- Example



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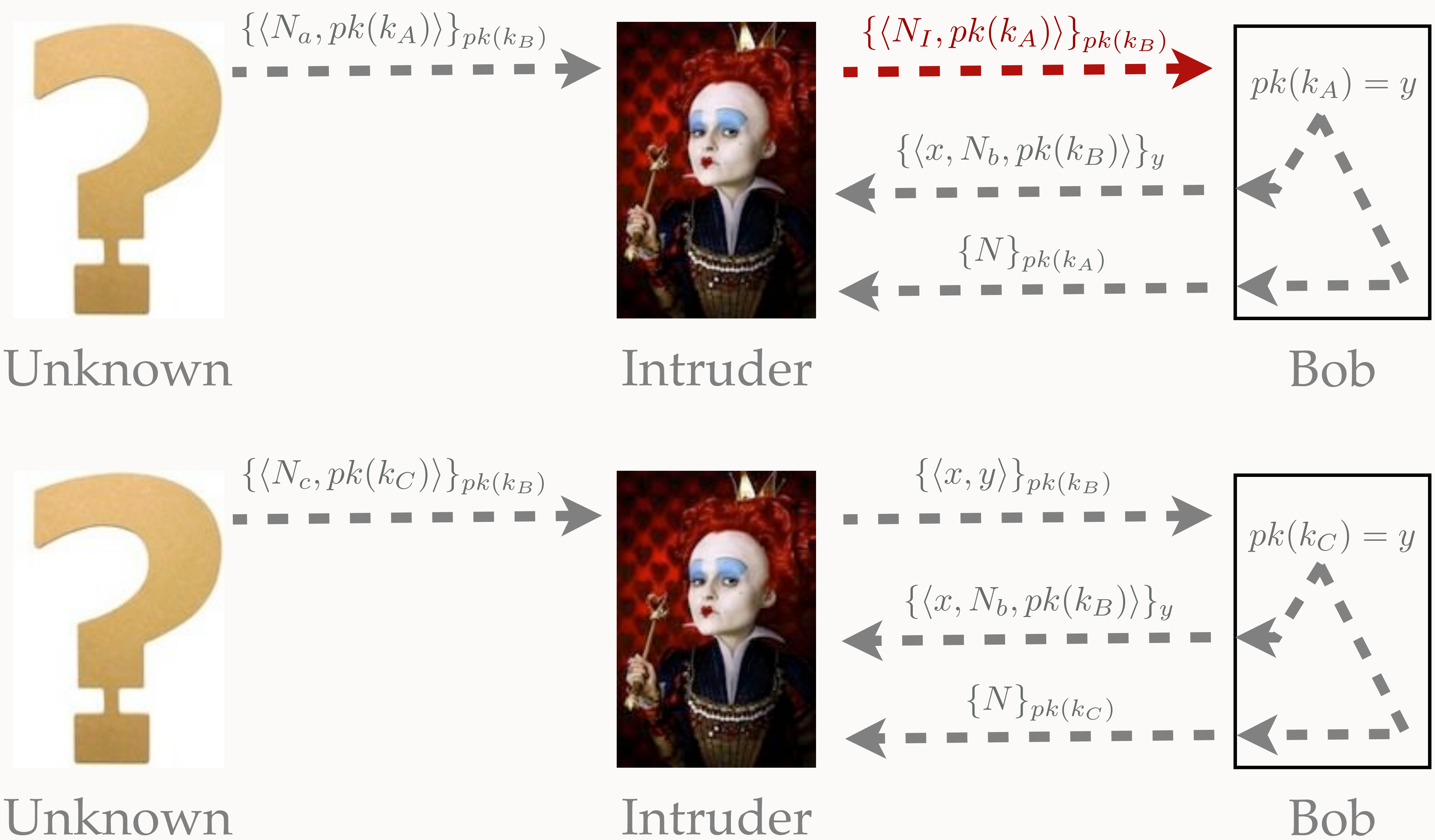

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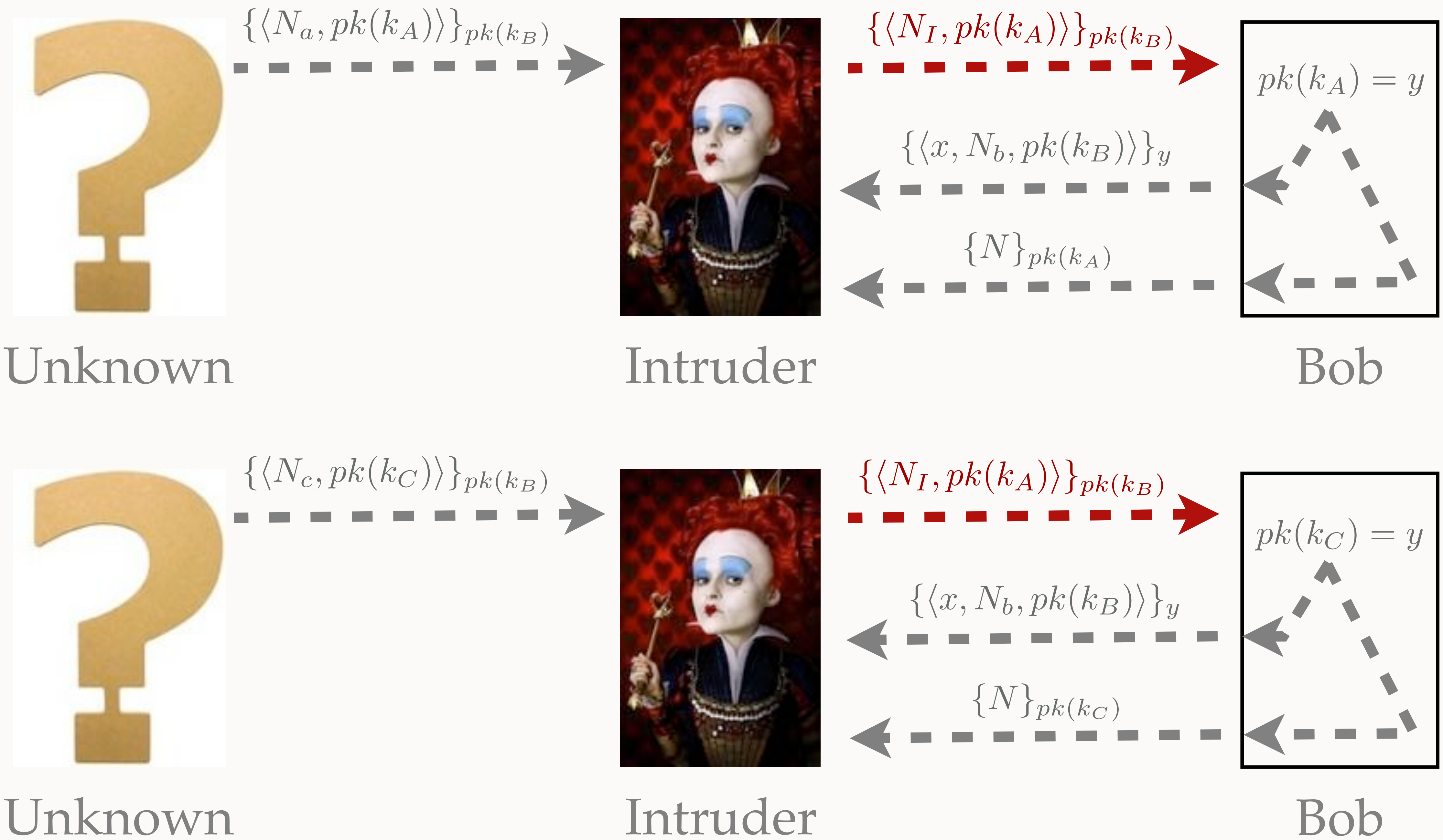

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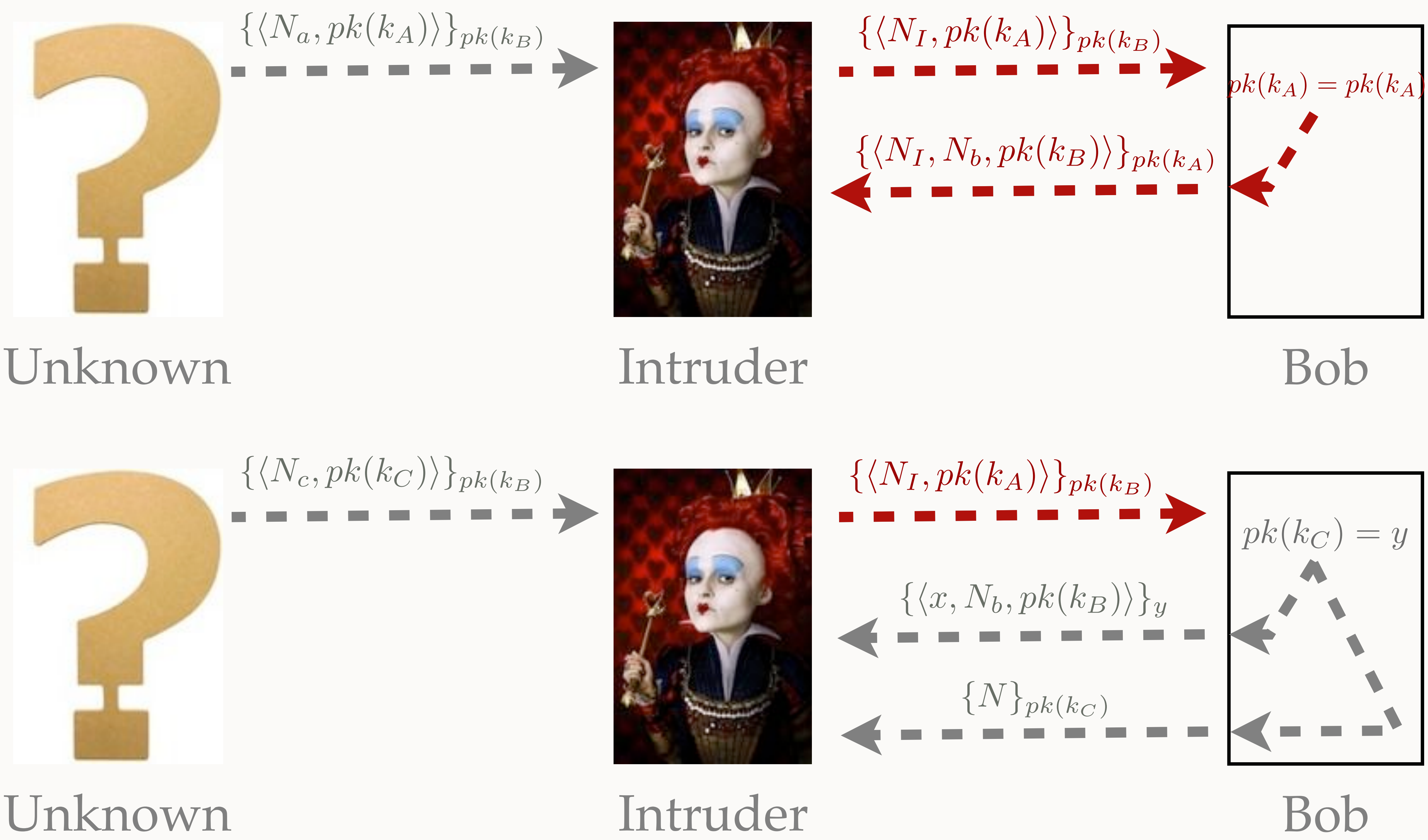

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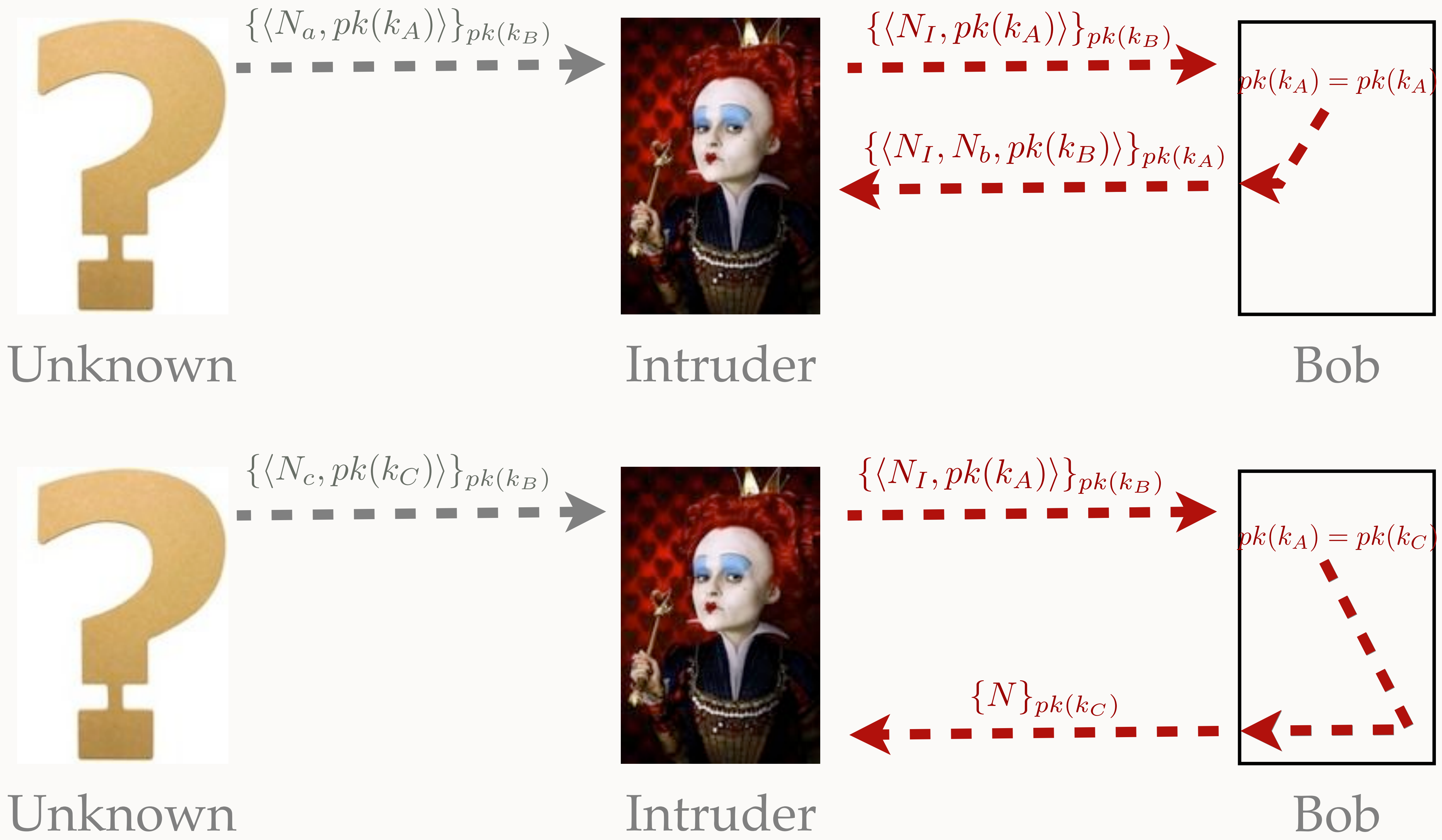

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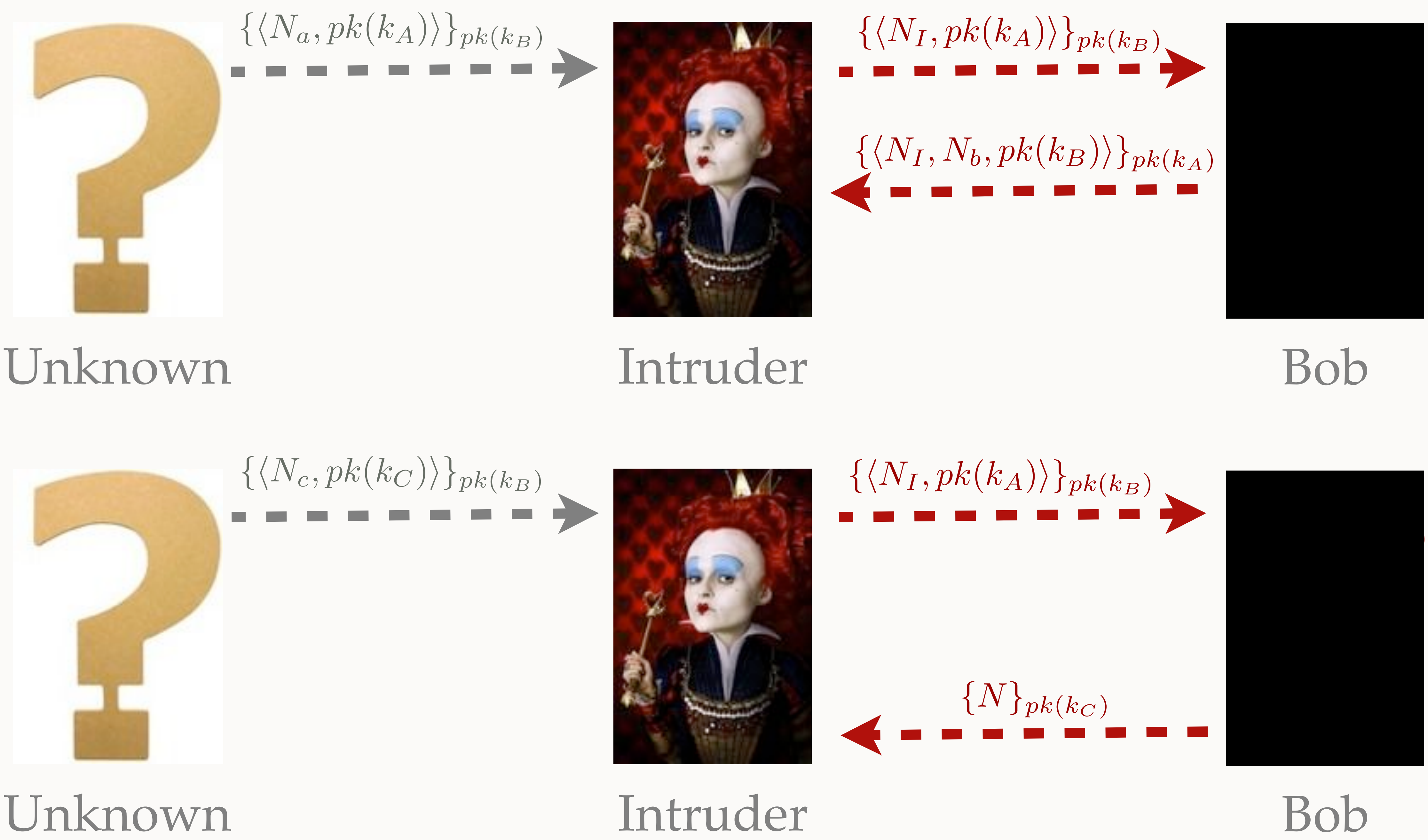

CONTRIBUTION

Decision procedure for trace equivalence

- Infinitely many traces are represented by symbolic constraint system

+ Protocol possibly non-determinist and with non trivial else branches

+ Private channels

- Finite set of cryptographic primitives : symmetric and asymmetric encryption, pairing and signature

- Bounded number of sessions (no replication in the process algebra)

CONSTRAINT SYSTEM

- One constraint system = one interleaving = several traces

Alice

Intruder

Bob

CONSTRAINT SYSTEM

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Alice

Intruder

Bob

$pk(k_A), pk(k_B), pk(k_C), N_I$

CONSTRAINT SYSTEM

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Alice

$\{\langle N_a, pk(k_A)\rangle\}_{pk(k_B)}$

Intruder

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CONSTRAINT SYSTEM

- One constraint system = one interleaving = several traces

$$pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A)\rangle\}_{pk(k_B)} \overset{?}{\vdash} \{\langle x, y\rangle\}_{pk(k_B)}$$

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$$y \overset{?}{=} pk(k_A)$$

CONSTRAINT SYSTEM

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$$pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A)\rangle\}_{pk(k_B)}, \{\langle x, N_b, pk(k_B)\rangle\}_y$$

$$y \overset{?}{=} pk(k_A)$$

CONSTRAINT SYSTEM

- One constraint system = one interleaving = several traces

$$D:\ pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A)\rangle\}_{pk(k_B)} \overset{?}{\vdash} \{\langle x, y\rangle\}_{pk(k_B)}$$

$$\Phi:\ pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A)\rangle\}_{pk(k_B)}, \{\langle x, N_b, pk(k_B)\rangle\}_y$$

$$E:\ y \overset{?}{=} pk(k_A)$$

CONSTRAINT SYSTEM

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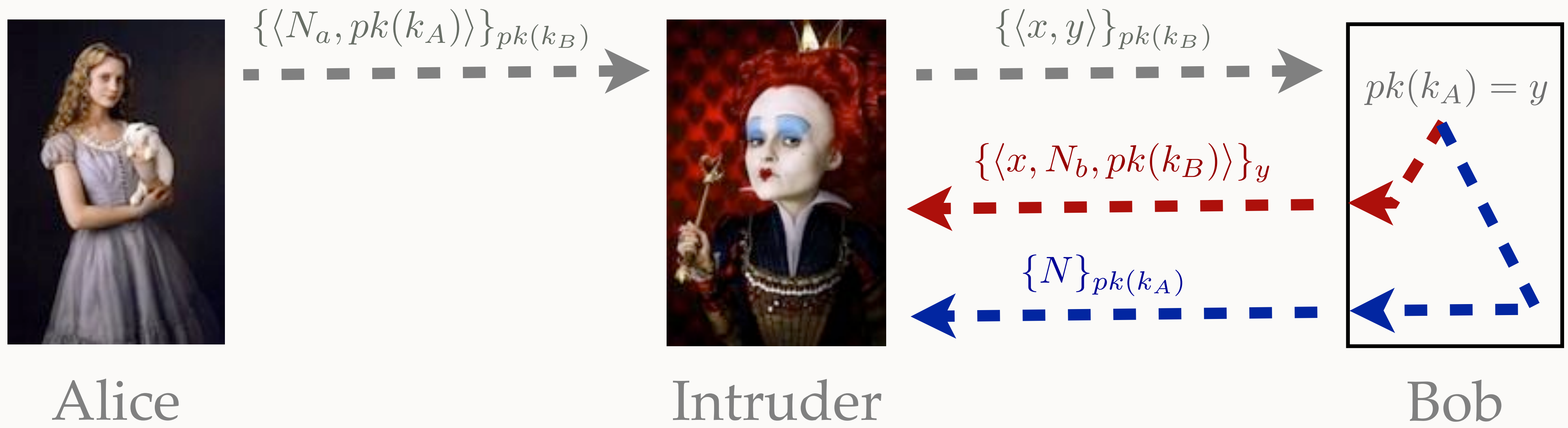
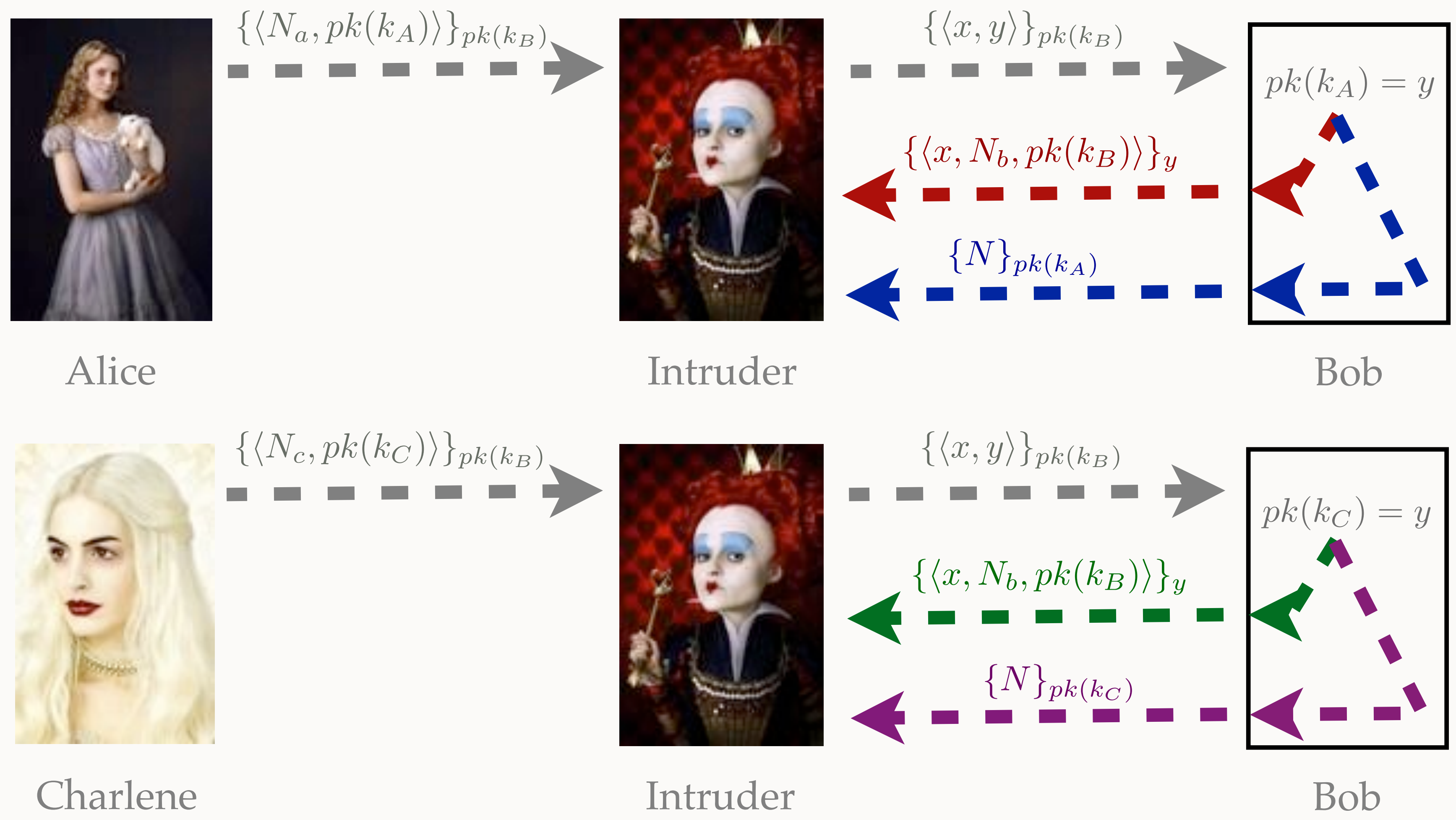


$$
\begin{aligned}
D &: \; pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A)\rangle\}_{pk(k_B)} \overset{?}{\vdash} \{\langle x, y\rangle\}_{pk(k_B)} \\
\Phi &: \; pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A)\rangle\}_{pk(k_B)}, \{\langle x, N_b, pk(k_B)\rangle\}_y \\
E &: \; y \overset{?}{=} pk(k_A)
\end{aligned}
$$

$$
\begin{aligned}
D &: \; pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A)\rangle\}_{pk(k_B)} \overset{?}{\vdash} \{\langle x, y\rangle\}_{pk(k_B)} \\
\Phi &: \; pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A)\rangle\}_{pk(k_B)}, \{N\}_{pk(k_A)} \\
E &: \; y \overset{?}{\neq} pk(k_A)
\end{aligned}
$$

CONSTRAINT SYSTEM

- Set of constraint systems



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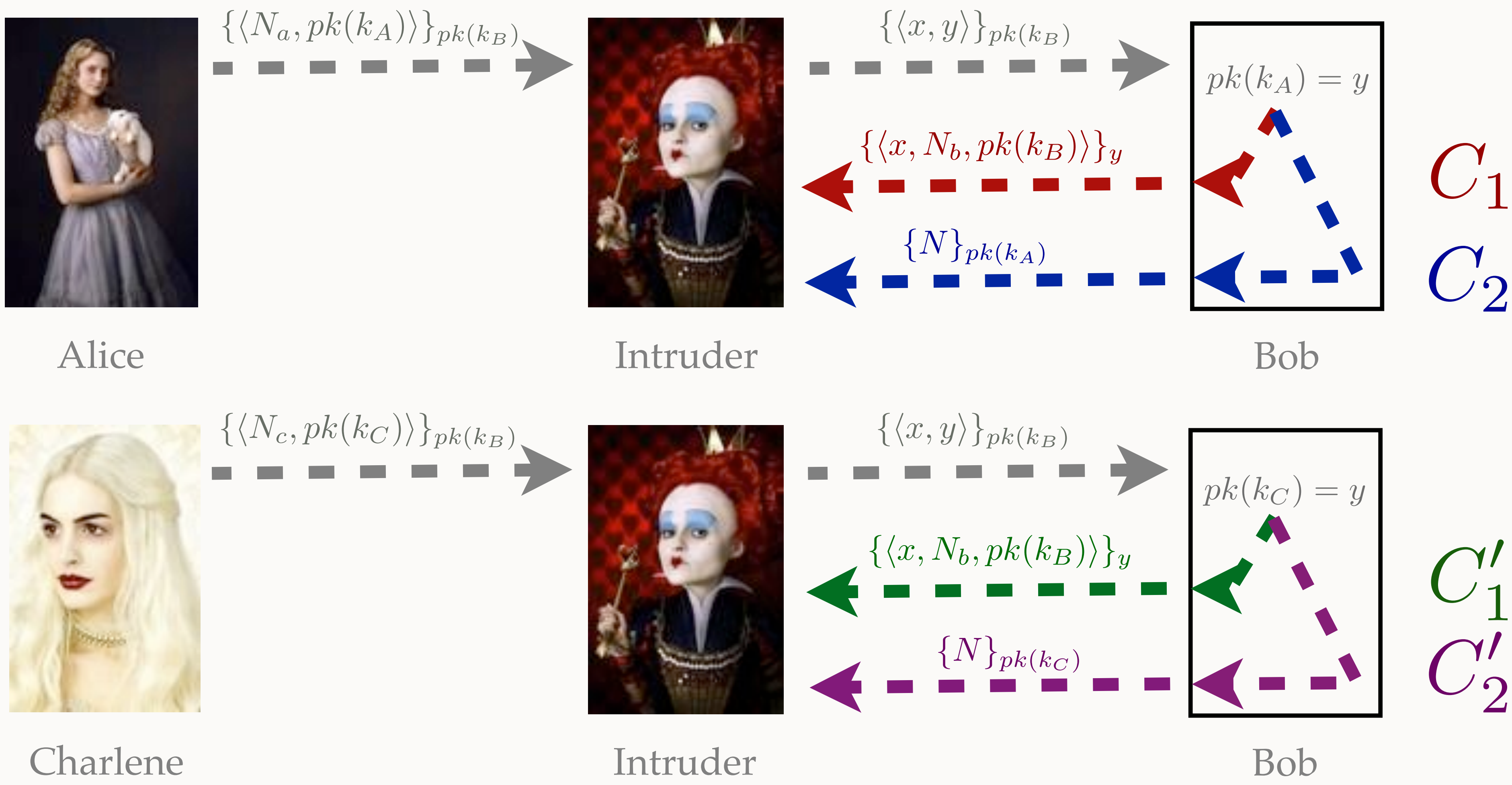

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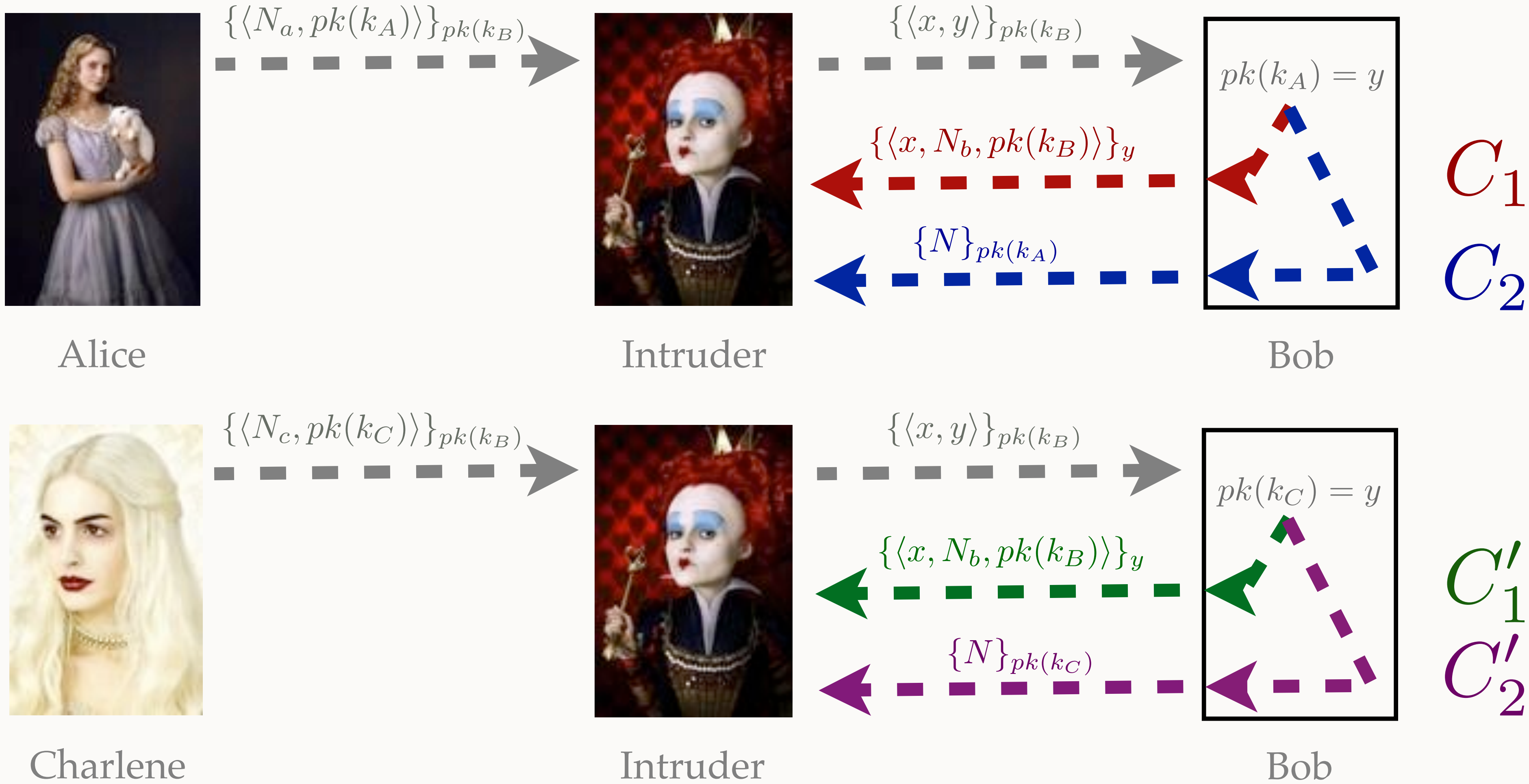

$$\{C_1;\ C_2\} \approx \{C'_1;\ C'_2\}$$

CONSTRAINT SYSTEM

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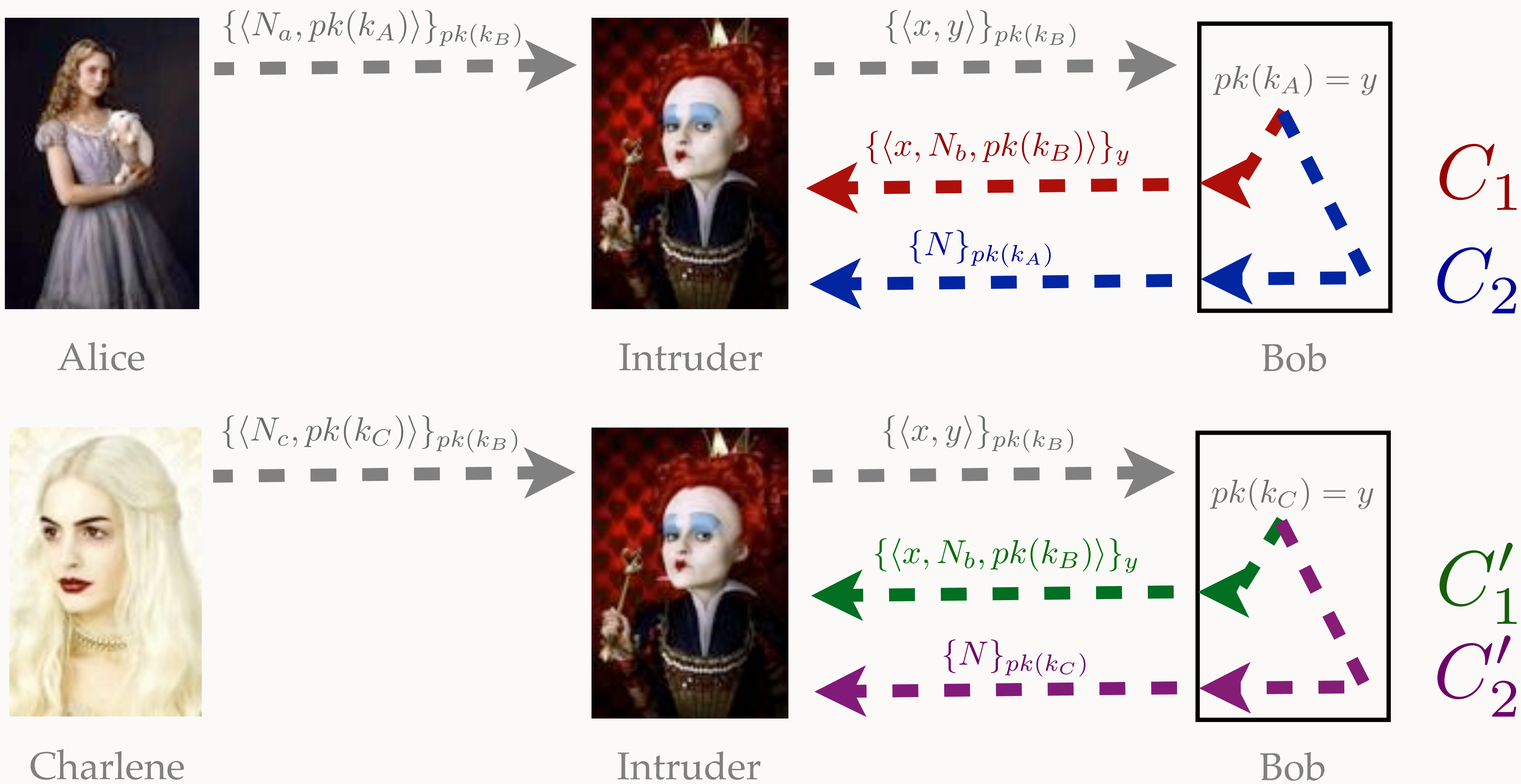


Symbolic equivalence between sets of constraint systems

CONSTRAINT SYSTEM

- Previous works on constraint system

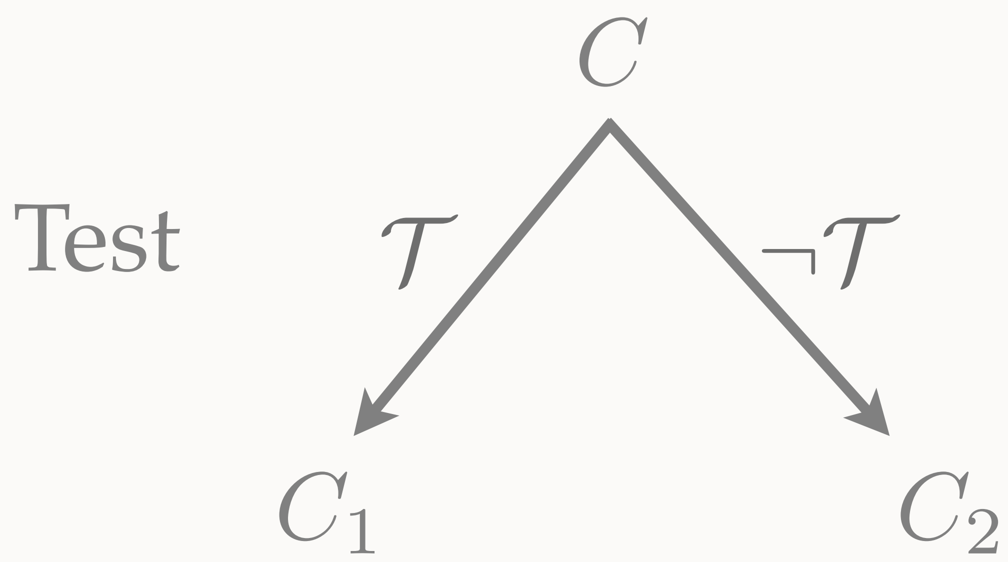
1. M. Baudet. *Sécurité des protocoles cryptographiques : aspects logiques et calculatoires.* Phd thesis
2. Y. Chevalier and M. Rusinowitch. *Decidability of equivalence of symbolic derivations.*
3. V. Cortier and S. Delaune. *A method for proving observational equivalence.*
4. A. Tiu and J. E. Dawson. *Automating open bisimulation checking for the spi calculus.*
5. V. Cheval, H. Comon-Lundh, S. Delaune. *Automating security analyss: symbolic equivalence of constraint systems*

Focus on :

- symbolic equivalence between two constraint systems (All)
- positive constraint system (no disequations) (All)
- subterm convergent equational theory (1,2 & 3)
- more restricted equational theory (4 & 5)

THE ALGORITHM

- Set of rules



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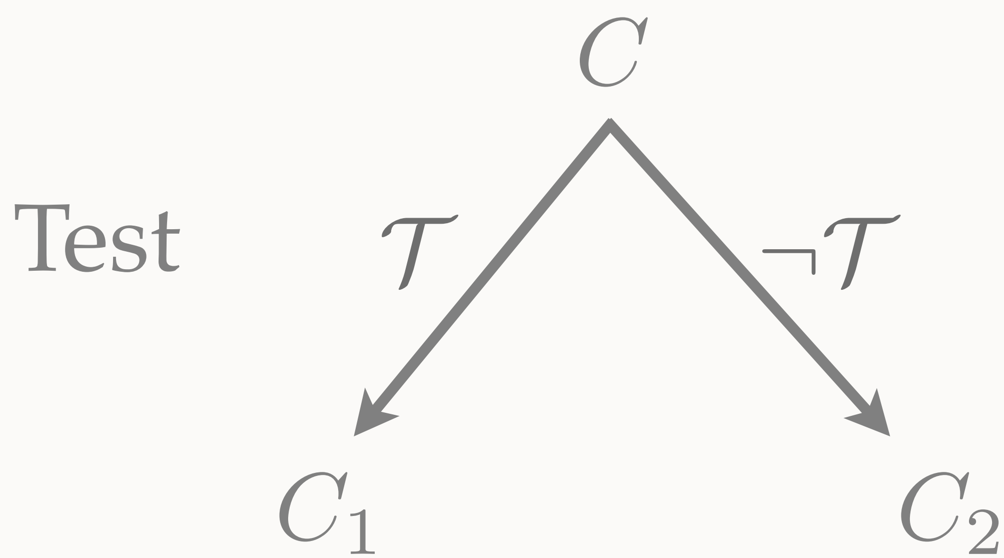
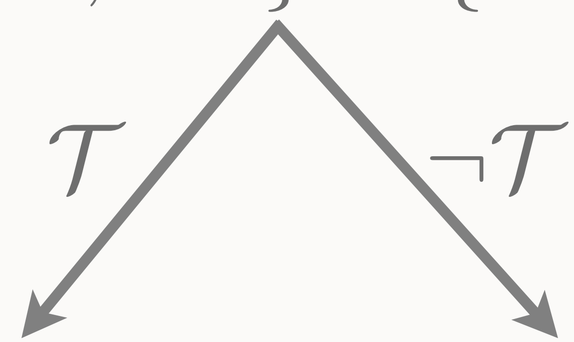


- How to apply the rules :

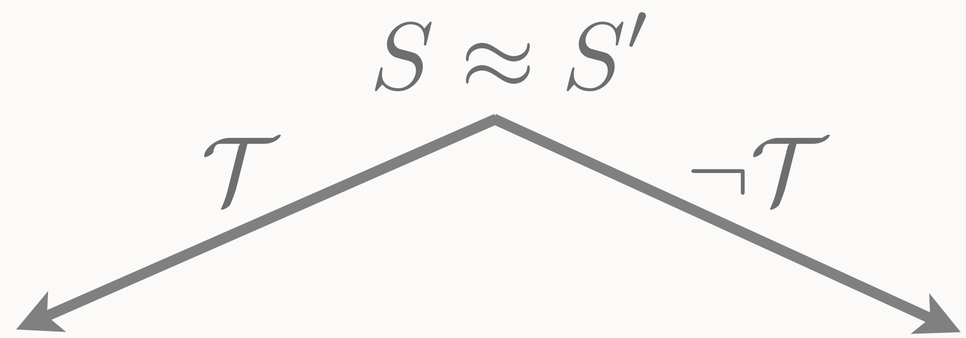
$$\{C^1;\ C^2;\ \ldots\} \approx \{C^n;\ \ldots\}$$



$$\{C_1^1;\ C_1^2;\ \ldots\} \approx \{C_1^n;\ \ldots\} \qquad \{C_2^1;\ C_2^2;\ \ldots\} \approx \{C_2^n;\ \ldots\}$$

THE ALGORITHM

- A complete execution



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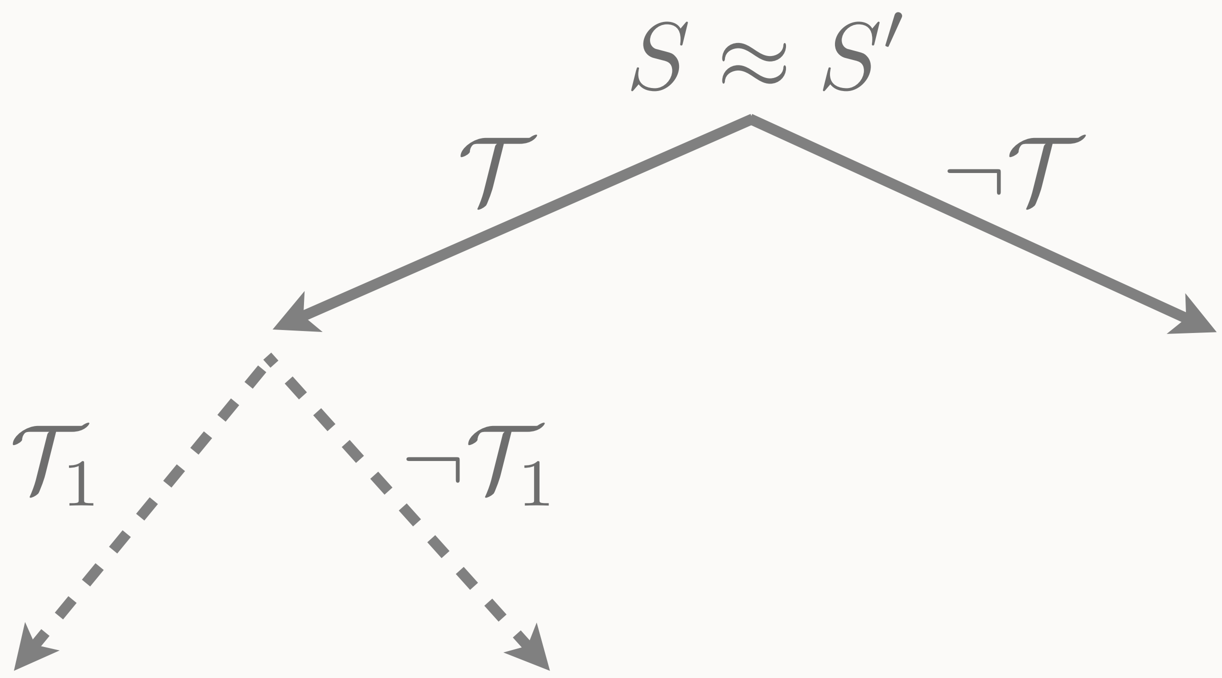

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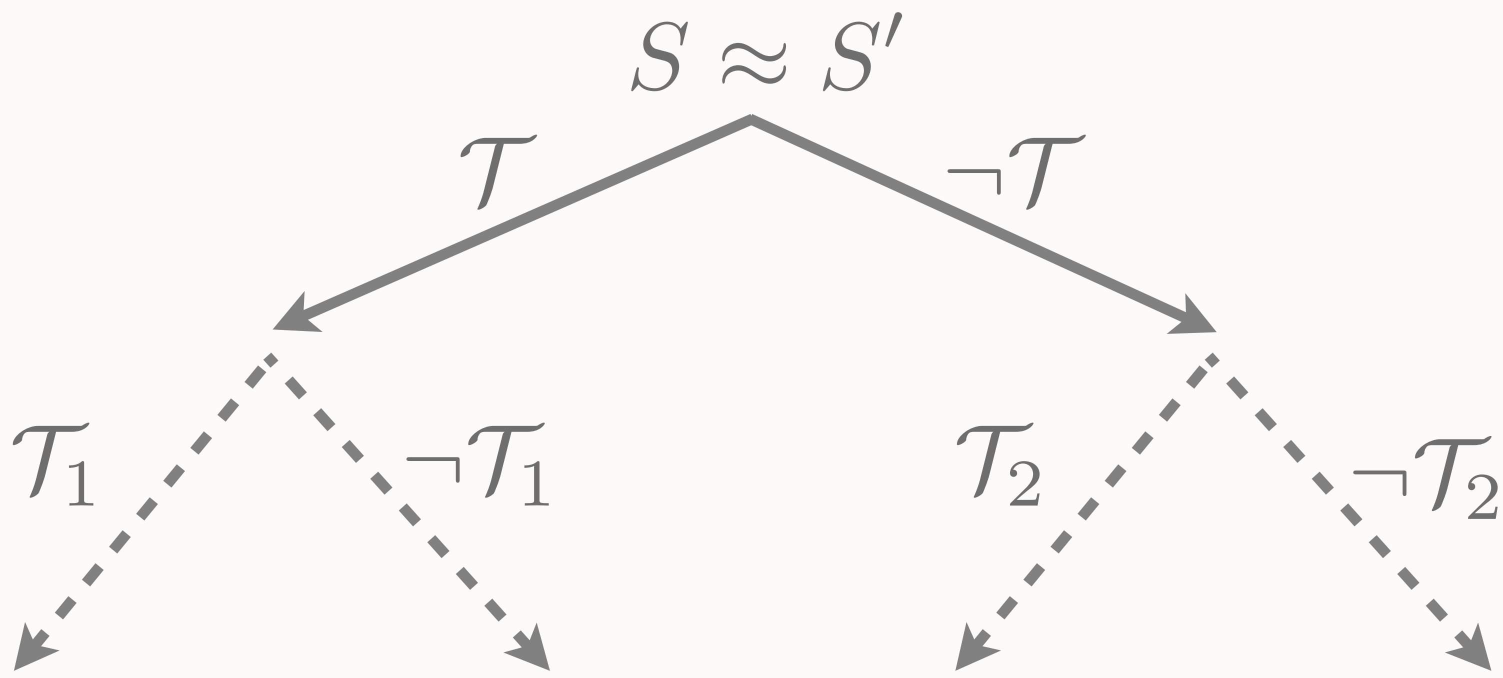

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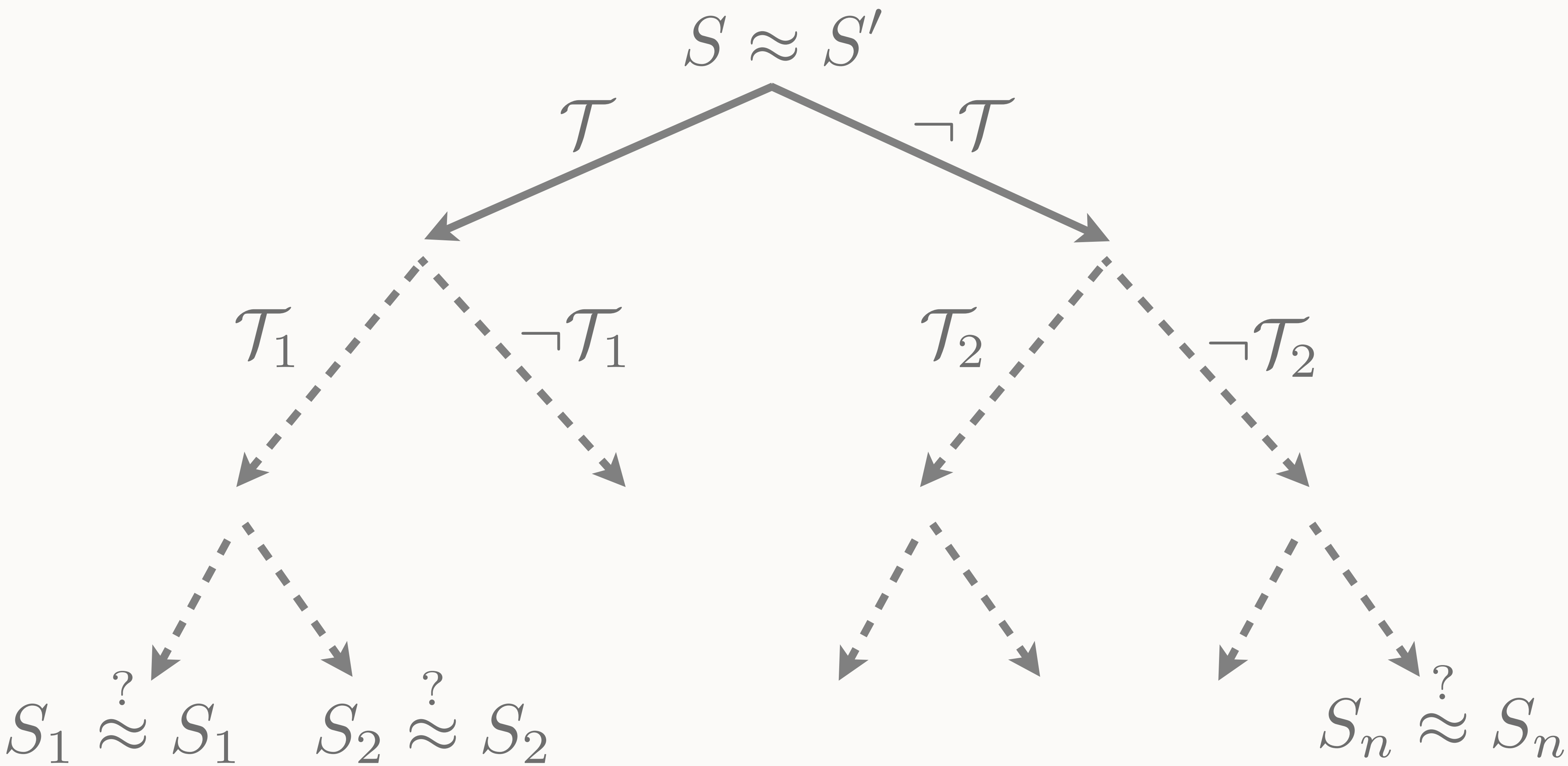
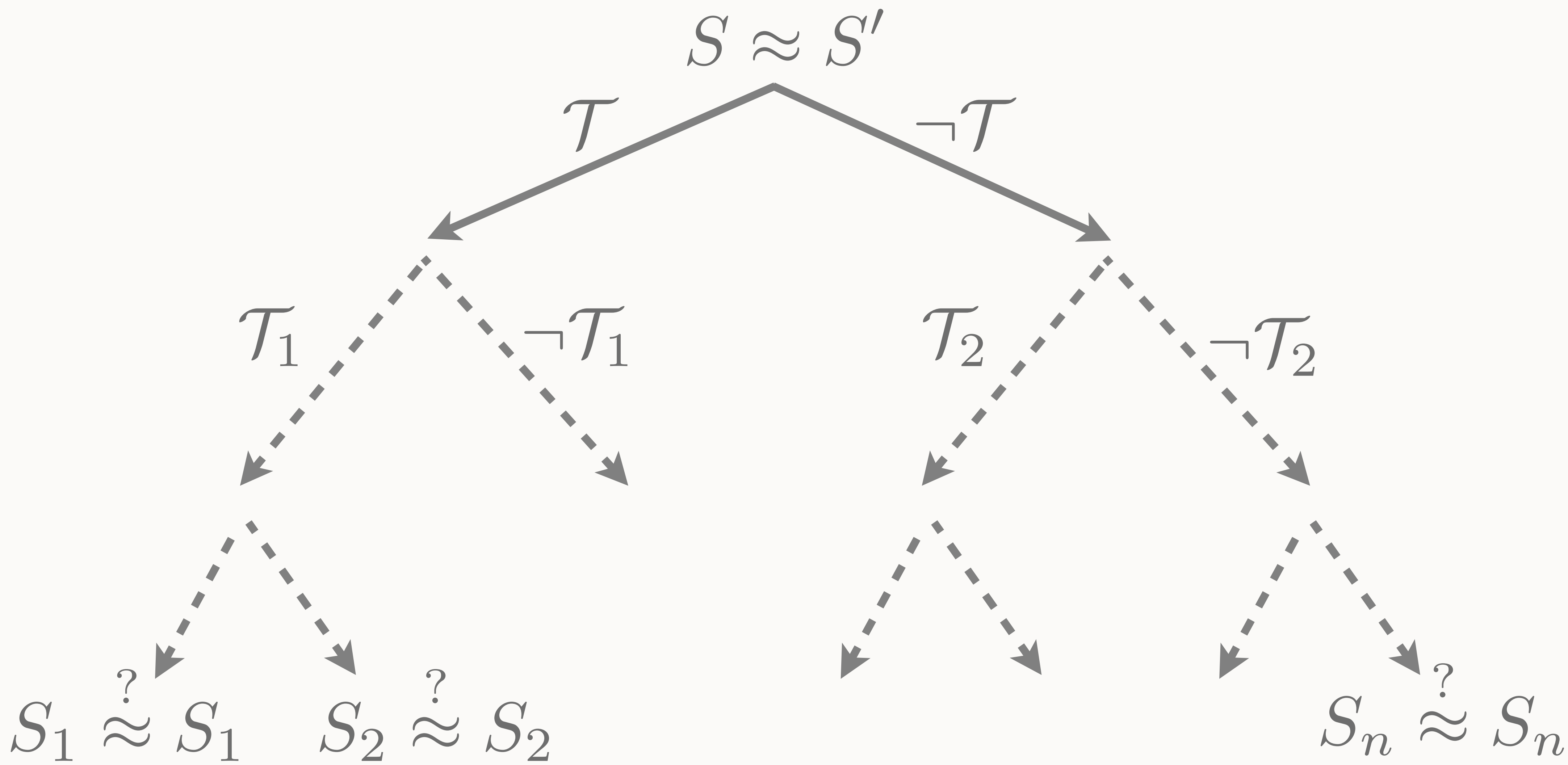


The application of the rules creates a binary tree where each node is a pair of sets of constraint systems

THE ALGORITHM

- A complete execution



The symbolic equivalence is syntactically decided on each leaf

THE ALGORITHM

- The solved form of a constraint system

  - Existence of solutions (Reachability)

$$m_1, \ldots, m_n \vdash x$$
$$m_1, \ldots, m_n, \ldots, m_{n'} \vdash y$$

  - Matching solutions (including disequations)

$$a, b \vdash x \quad a, b, c \vdash y \quad x \neq y$$

$$a, b \vdash x \quad a, b, c \vdash y \quad x \neq f(y)$$

  - Static equivalence

$$a, \{b\}_c \vdash x \quad a, \{b\}_c, c \vdash y$$

$$a, b \vdash x \quad a, b, c \vdash y$$

RESULT

Let (S_0, S'_0) be an initial pair of set of constraint systems, we have :

(S, S')

(S, S')

RESULT

Let (S_0, S'_0) be an initial pair of set of constraint systems, we have :

If all leaves (S, S') on the tree satisfy the testing condition then $S_0 \approx S'_0$.

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If $S_0 \approx S'_0$ then all leaves (S, S') on the tree satisfy the testing condition.

RESULT

Let (S_0, S_0') be an initial pair of set of constraint systems, we have :

If all leaves (S, S') on the tree satisfy the testing condition then $S_0 \approx S_0'$.

If $S_0 \approx S_0'$ then all leaves (S, S') on the tree satisfy the testing condition.

The strategy terminates

FUTURE WORK

- Contribution

Decision procedure for trace equivalence

- Infinitely many traces are represented by symbolic constraint system

+ Protocol possibly non-determinate and with non trivial else branches

+ Private channels

- Finite set of cryptographic primitives : symmetric and asymmetric encryption, pairing and signature

- Bounded number of sessions (no replication in the process algebra)

- Future work
  - Efficient implementation (application on more case studies)
  - More cryptographic primitives
  - Link with ProVerif

TERMINATION

- The disequations problem

$$
\begin{array}{rl}
 & a, b \vdash x_1 \\
D: & a, b \vdash x_2 \\
 & a, b \vdash y \\
E: & [x_1 \neq y \vee x_2 \neq a] \wedge y \neq \langle x_1, x_2, b \rangle
\end{array}
$$

TERMINATION

- The disequations problem

$$E : [x_1 \neq y \vee x_2 \neq a] \wedge y \neq \langle x_1, x_2, b \rangle$$

TERMINATION

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TERMINATION

- The disequations problem

$$E : [x_1 \neq y \vee x_2 \neq a] \wedge y \neq \langle x_1, x_2, b \rangle$$

$$x_2 = a$$

$$E : [x_1 \neq y] \wedge y \neq \langle x_1, a, b \rangle$$

TERMINATION

- The disequations problem

$$E : [x_1 \neq y \vee x_2 \neq a] \wedge y \neq \langle x_1, x_2, b \rangle$$

$x_2 = a$

$$E : [x_1 \neq y] \wedge y \neq \langle x_1, a, b \rangle$$

$y = \langle y_1, y_2, y_3 \rangle$

TERMINATION

- The disequations problem

$$E : [x_1 \neq y \vee x_2 \neq a] \wedge y \neq \langle x_1, x_2, b \rangle$$

$$\downarrow \; x_2 = a$$

$$E : [x_1 \neq y] \wedge y \neq \langle x_1, a, b \rangle$$

$$\downarrow \; y = \langle y_1, y_2, y_3 \rangle$$

$$E : [x_1 \neq \langle y_1, y_2, y_3 \rangle] \wedge \langle y_1, y_2, y_3 \rangle \neq \langle x_1, a, b \rangle$$

TERMINATION

- The disequations problem

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$$E : [x_1 \neq y] \wedge y \neq \langle x_1, a, b \rangle$$

$$y = \langle y_1, y_2, y_3 \rangle$$

$$E : [x_1 \neq \langle y_1, y_2, y_3 \rangle] \wedge \langle y_1, y_2, y_3 \rangle \neq \langle x_1, a, b \rangle$$

TERMINATION

- The disequations problem

$$E : [x_1 \neq y \vee x_2 \neq a] \wedge y \neq \langle x_1, x_2, b\rangle$$

$$x_2 = a$$

$$E : [x_1 \neq y] \wedge y \neq \langle x_1, a, b\rangle$$

$$y = \langle y_1, y_2, y_3\rangle$$

$$E : [x_1 \neq \langle y_1, y_2, y_3\rangle] \wedge \langle y_1, y_2, y_3\rangle \neq \langle x_1, a, b\rangle$$

$$E : [x_1 \neq \langle y_1, y_2, y_3\rangle] \wedge [y_1 \neq x_1 \vee y_2 \neq a \vee y_3 \neq b]\rangle$$

TERMINATION

- The disequations problem

$$E : [x_1 \neq y \vee x_2 \neq a] \wedge y \neq \langle x_1, x_2, b \rangle$$

$$\downarrow \; x_2 = a$$

$$E : [x_1 \neq y] \wedge y \neq \langle x_1, a, b \rangle$$

$$\downarrow \; y = \langle y_1, y_2, y_3 \rangle$$

$$E : [x_1 \neq \langle y_1, y_2, y_3 \rangle] \wedge \langle y_1, y_2, y_3 \rangle \neq \langle x_1, a, b \rangle$$

$$\downarrow$$

$$E : [x_1 \neq \langle y_1, y_2, y_3 \rangle] \wedge [y_1 \neq x_1 \vee y_2 \neq a \vee y_3 \neq b]\rangle$$

$$\downarrow \; y_3 = b$$

TERMINATION

- The disequations problem

$$E : [x_1 \neq y \vee x_2 \neq a] \wedge y \neq \langle x_1, x_2, b \rangle$$

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$$E : [x_1 \neq \langle y_1, y_2, y_3 \rangle] \wedge [y_1 \neq x_1 \vee y_2 \neq a \vee y_3 \neq b] \rangle$$

$$y_3 = b$$

$$E : [x_1 \neq \langle y_1, y_2, b \rangle] \wedge [y_1 \neq x_1 \vee y_2 \neq a] \rangle$$